EE 2240

Problem #12

For the system described by $\ddot{x}+12\dot{x}+36x=0$:

a. Determine the characteristic equation.

$$r^2+12r+36=0$$

b. Determine the natural frequency, ω_n.

$$\omega_n^2=36 \Rightarrow \omega_n=6$$

c. Determine the damping ratio, ζ.

$$2\zeta\omega_n=12 \Rightarrow \zeta=\frac{12}{2\omega_n}=1$$

d. Determine the numerical values of the two roots of the characteristic equation.

$$r^2+12r+36=(r+6)^2 \Rightarrow r=-6,-6$$

e. Classify the system as *overdamped*, *critically damped*, *underdamped*, or *undamped*.

The system is critically damped ($\zeta=1$)

f. Assuming $x(0)=1$ and $\dot{x}(0)=-3$, determine the solution of the given equation.

$$x(t)=(A+Bt)e^{-6t}$$

$$\dot{x}(t)=Be^{-6t}-6(A+Bt)e^{-6t}$$

$$\left.\begin{aligned} x(0)&=A=1 \\ \dot{x}(0)&=B-6A=-3 \end{aligned}\right\} \quad \begin{aligned} A&=1 \\ B&=3 \end{aligned}$$

$$\therefore\ x(t)=(1+3t)e^{-6t}, \quad t\geq 0$$